Name: Key Date:

Pre-Calculus 11 Ch3/4 HW Lesson 8: Solving Quadratic Equations by CTS

1. When solving the equation $9 = x^2$, how many solutions will there be? Explain:

   THERE WILL BE TWO ANSWERS. B/C $(3)^2=9$ AND $(-3)^2=9$ SO $x=\pm 3$

2. When Solving the equation $12 = (x-3)^2$, how many solutions will there be? What are they?

   $12=(x-3)^2$

   $\pm\sqrt{12} = x-3$

   $3\pm\sqrt{12} = x$.

   $\therefore x_1 = 3+\sqrt{12}$ or $x_1 = 3+2\sqrt{3}$

   $x_2 = 3-\sqrt{12}$ or $x_2 = 3-2\sqrt{3}$

3. What are we looking for on a graph when solving for "x"?

   WHEN SOLVING FOR 'x' IN AN EQUATION, ONE SIDE IS EQUAL TO ZERO. THIS MEANS THE y-COORDINATE OF THE GRAPH IS ZERO. $\therefore$ WE ARE LOOKING FOR THE x-INTERCEPTS.

4. Suppose we solve for "x" and there is only one answer. What does this mean?

   - IF IT'S AN QUADRATIC EQUATION AND THERE'S ONLY ONE ANSWER, THAT MEANS THERE IS ONLY ONE x-INTERCEPT.
   - A PARABOLA WITH ONLY ONE x-INTERCEPT MEANS THE VERTEX IS ON THE x-AXIS.

5. Solve each of the following equations algebraically:

a) $(x-3)^2-12=0$ $(x-3)^2=12$ $x-3=\pm 2\sqrt{3}$ $x=3\pm 2\sqrt{3}$	b) $(2x+4)^2-16=0$ $(2x+4)^2=16$ $2x+4=\pm 4$ $2x=-4\pm 4$ $x=\frac{-4+4}{2}$ or $\frac{-4-4}{2}$ $x_1=0$ $x_2=-4$	c) $-4(x+7)^2+14=0$ $-4(x+7)^2=-14$ $(x+7)^2=\frac{14}{4}$ $x+7=\pm\sqrt{\frac{7}{2}}$ $x=-7\pm\sqrt{\frac{7}{2}}$ $x_1=-7+\sqrt{\frac{7}{2}}$ or $x_2=-7-\sqrt{\frac{7}{2}}$
d) $0.5(x+11)^2-11=0$ $0.5(x+11)^2=11$ $(x+11)^2=22$ $x+11=\pm\sqrt{22}$ $x=-11\pm\sqrt{22}$ $x_1=-11+\sqrt{22}$ or $x_2=-11-\sqrt{22}$	e) $(x+5)^2+12=0$ $(x+5)^2=-12$ $x+5=\sqrt{-12}$ CAN'T SQUARE ROOT A NEGATIVE VALUE. $\therefore$, NO REAL ANSWER.	f) $\frac{(2x+1)^2}{3}-15=0$ $\frac{(2x+1)^2}{3}=15$ $(2x+1)^2=45$ $2x+1=\pm\sqrt{45}$ $2x=-1\pm 3\sqrt{5}$ $x=\frac{-1\pm 3\sqrt{5}}{2}$ $x_1=-\frac{1}{2}+\frac{3}{2}\sqrt{5}$ $x_2=-\frac{1}{2}-\frac{3}{2}\sqrt{5}$ NOTE $\sqrt{45}=\sqrt{9}\times\sqrt{5}$ $=3\sqrt{5}$

g) $-\frac{2}{3}\left(x-\frac{3}{2}\right)^2+4=0$ $-\frac{2}{3}(x-\frac{3}{2})^2=-4$ $(x-\frac{3}{2})^2=-4\times(-\frac{3}{2})$ $(x-\frac{3}{2})^2=6$ $x-\frac{3}{2}=\pm\sqrt{6}$ $x=\frac{3}{2}\pm\sqrt{6}$ $x_1=\frac{3}{2}+\sqrt{6}$ $x_2=\frac{3}{2}-\sqrt{6}$	h) $-\frac{7}{3}(2x-13)^2+15=0$ $-\frac{7}{3}(2x-13)^2=-15$ $(2x-13)^2=-15\times(-\frac{3}{7})$ $(2x-13)^2=\frac{45}{7}$ $2x-13=\pm\sqrt{\frac{45}{7}}$ $2x=13\pm\sqrt{\frac{45}{7}}$ $x=\frac{13}{2}\pm\frac{1}{2}\sqrt{\frac{45}{7}}$ $x_1=\frac{13}{2}+\frac{3}{2}\sqrt{\frac{5}{7}}$ $x_2=\frac{13}{2}-\frac{3}{2}\sqrt{\frac{5}{7}}$	i) $\frac{17}{3}(2x-21)^2=0$ $\frac{17}{3}(2x-21)^2=0$ Since $\frac{17}{3}\times 0=0$ So $(2x-21)=0$ $2x=21$ $x=\frac{21}{2}$

6. Solve each of the following quadratic equations by Completing the Square. Please show all your steps:

a) $0=3x^2+8x-5$ $0=(3x^2+8x)-5$ $0=3(x^2+\frac{8}{3}x)-5$ $0=3(x^2+\frac{8}{3}x+\frac{16}{9}-\frac{16}{9})-5$ $0=3(x^2+\frac{8}{3}x+\frac{16}{9})-\frac{16}{3}-5$ $0=3(x-\frac{4}{3})^2-\frac{16}{3}-\frac{15}{3}$ $0=3(x-\frac{4}{3})^2-\frac{31}{3}$ ← C.T.S. $\frac{31}{3}=3(x-\frac{4}{3})^2$ $\frac{31}{9}=(x-\frac{4}{3})^2$ $\pm\frac{\sqrt{31}}{3}=x-\frac{4}{3}$ $\frac{4}{3}\pm\frac{\sqrt{31}}{3}=x$ $x_1=\frac{4+\sqrt{31}}{3}$ $x_2=\frac{4-\sqrt{31}}{3}$	b) $0=4x^2+12x-11$ $0=4(x^2+3x)-11$ $0=4(x^2+3x+\frac{9}{4}-\frac{9}{4})-11$ $0=4(x^2+3x+\frac{9}{4})-9-11$ $0=4(x+\frac{3}{2})^2-20$ $20=4(x+\frac{3}{2})^2$ $5=(x+\frac{3}{2})^2$ $\pm\sqrt{5}=x+\frac{3}{2}$ $-\frac{3}{2}\pm\sqrt{5}=x$ $x_1=-\frac{3}{2}+\sqrt{5}$ $x_2=-\frac{3}{2}-\sqrt{5}$
c) $4x^2=2-13x$ $4x^2+13x-2=0$ $4(x^2+\frac{13}{4}x)-2=0$ $4(x^2+\frac{13}{4}x+\frac{169}{64}-\frac{169}{64})-2=0$ $4(x^2+\frac{13}{4}x+\frac{169}{4})-\frac{169}{16}-2=0$ $4(x+\frac{13}{2})^2=\frac{169}{16}+\frac{32}{16}$ $4(x+\frac{13}{2})^2=\frac{201}{16}$ $(x+\frac{13}{2})^2=\frac{201}{64}$ $x+\frac{13}{2}=\pm\sqrt{\frac{201}{64}}$ $x=-\frac{13}{2}\pm\frac{\sqrt{201}}{8}$ $x_1=-\frac{13}{2}+\frac{\sqrt{201}}{8}$ $x_2=-\frac{13}{2}-\frac{\sqrt{201}}{8}$	d) $0=-5x^2+10x-3$ $0=-5(x^2-2x)-3$ $0=-5(x^2-2x+1-1)-3$ $0=-5(x^2-2x+1)+5-3$ $0=-5(x-1)^2+2$ $-2=-5(x-1)^2$ $\frac{2}{5}=(x-1)^2$ $\pm\sqrt{\frac{2}{5}}=x-1$ $1\pm\sqrt{\frac{2}{5}}=x$ $x_1=1+\sqrt{\frac{2}{5}}$ $x_2=1-\sqrt{\frac{2}{5}}$

7. The equation of a parabola is given by the equation: $y = 3x^2 + 5x - 10$. Find the roots [aka: coordinates of the x-intercepts] by completing the square:

$0 = (3x^2 + 5x) - 10$

$0 = 3(x^2 + \frac{5}{3}x) - 10$

$0 = 3(x^2 + \frac{5}{3}x + \frac{25}{36} - \frac{25}{36}) - 10$

$0 = 3(x^2 + \frac{5}{3}x + \frac{25}{36}) - \frac{25}{12} - 10$

$0 = 3(x + \frac{5}{6})^2 - \frac{25}{12} - \frac{120}{12}$

$\frac{145}{12} = 3(x + \frac{5}{6})^2$

$\frac{145}{36} = (x + \frac{5}{6})^2$

$\pm\sqrt{\frac{145}{36}} = x + \frac{5}{6}$

$-\frac{5}{6} \pm \frac{\sqrt{145}}{6} = x$

$x_1 = \left(-\frac{5}{6} + \frac{\sqrt{145}}{6}, 0\right)$ //

$x_2 = \left(-\frac{5}{6} - \frac{\sqrt{145}}{6}, 0\right)$ //

8. A rocket is shot into the sky and the height of the rocket is given by the equation: $h(t) = -5t^2 + 12t + 10$ where "t" is the number of seconds after the rocket was launched.

   a. What is the height when the rocket hits the ground? AT THE GROUND, $h = 0$ //

   b. At what time does the rocket hit the ground?

① FIND 't' SUCH THAT $h(t) = 0$.

$0 = -5t^2 + 12t + 10$

$0 = -5(t^2 - \frac{12}{5}t) + 10$

$0 = -5(t^2 - \frac{12}{5}t + \frac{36}{25} - \frac{36}{25}) + 10$

$0 = -5(t^2 - \frac{12}{5}t + \frac{36}{25}) + \frac{36}{5} + \frac{50}{5}$

$-\frac{86}{5} = -5(t - \frac{6}{5})^2$

$\frac{86}{25} = (t - \frac{6}{5})^2$

$\pm\frac{\sqrt{86}}{5} = t - \frac{6}{5}$

$\frac{6}{5} \pm \frac{\sqrt{86}}{5} = t$

$t_1 = \frac{6}{5} - \frac{\sqrt{86}}{5}$ THIS IS NEGATIVE, SO CAN'T TAKE THIS ONE

$\boxed{t_2 = \frac{6}{5} + \frac{\sqrt{86}}{5}}$ //

   c. After how many seconds will the rocket be at a height of 30meters?

① FIND 't' WHEN $h(t) = 30$

$30 = -5t^2 + 12t + 10$

$30 = -5(t^2 - \frac{12t}{5}) + 10$

$30 = -5(t^2 - \frac{12t}{5} + \frac{36}{25} - \frac{36}{25}) + 10$

$20 = -5(t^2 - \frac{12t}{5} + \frac{36}{25}) + \frac{36}{5}$

$20 - \frac{36}{5} = -5(t - \frac{6}{5})^2$

$\frac{100}{5} - \frac{36}{5} = -5(t - \frac{6}{5})^2$

$\frac{64}{5} = -5(t - \frac{6}{5})^2$

$-\frac{64}{25} = (t - \frac{6}{5})^2$

$\pm\sqrt{-\frac{64}{25}} = t - \frac{6}{5}$

CAN'T SQUARE ROOT A NEGATIVE VALUE, SO NO SOLN.

THE ROCKET WILL NEVER REACH A HEIGHT OF 30 m.

9. The sum of an arithmetic series is given by the equation: $S = \frac{n}{2}(2 \times a + [n-1]d)$, where "n" is the number of terms, "a" is the first term, and "d" is the common difference. If the first term *"a"* is 10, common difference *"d"* is 4, and the sum *"S"* is 1144, find the number of terms *"n"* in the series.

$a = 10$

$d = 4$

$S = 1144$

$n = ?$

$1144 = \frac{n}{2}[2(10) + (n-1)4]$

$2288 = n(20 + 4n - 4)$

$2288 = n(16 + 4n)$

$2288 = 4n^2 + 16n$

$0 = 4n^2 + 16n - 2288$

$0 = n^2 + 4n - 572$

$0 = (n + 26)(n - 22)$

$572 = 4 \times 11 \times 13$

$= 2 \times 2 \times 11 \times 13$

$= 22 \times 26$

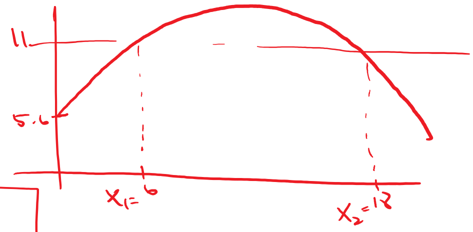
$4\overline{)572}$ 143

∴ $n \neq -26$ CAN'T HAVE A NEGATIVE NUMBER OF TERMS

$n = 22$

∴ THERE ARE 22 TERMS //

10. On desmos, the formula for a perfect basketball shot is given by the formula: $h(x) = -0.05x^2 + 1.2x + 5.6$, where "h" is the height of the ball and "x" is the distance from the shooter. How far is the ball from the shooter when the height of the ball is 11feet high?
https://www.desmos.com/calculator/djikpphgde



① $h(x) = 11$

$11 = -0.05x^2 + 1.2x + 5.6$

$11 = -0.05(x^2 - 24x) + 5.6$

$11 = \frac{-1}{20}(x^2 - 24x + 144 - 144) + 5.6$

$11 - 5.6 = \frac{-1}{20}(x^2 - 24x + 144) + \frac{144}{20}$

$5.4 - 7.2 = \frac{-1}{20}(x-12)^2$

$-1.8 = \frac{-1}{20}(x-12)^2$

$36 = (x-12)$

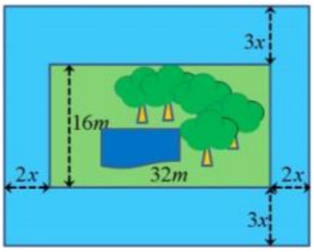
$36 = (x-12)^2$

$\pm 6 = x - 12$

$12 \pm 6 = x$

$\boxed{x_1 = 6 \qquad x_2 = 18}$

11. A rectangular playground (16m by 32m) has a walkway around it as shown below. If adding the walkway doubles the area of the playground, find the value of "x":



① AREA OF GARDEN $= 16 \times 32$

② AREA of GARDEN + PATHWAY $= 2 \times 16 \times 32$ (AREA IS DOUBLED)

$(32+4x)(16+6x) = 2 \times 16 \times 32$

$\cancel{4}(8+x)\cancel{2}(8+3x) = \cancel{2} \times \cancel{16} \times 32$

$(8+x)(8+3x) = 128$

$64 + 8x + 24x + 3x^2 = 128$

$3x^2 + 32x - 64 = 0$

$3(x^2 + \frac{32}{3}x) - 64 = 0$

$3(x^2 + \frac{32}{3}x + \frac{256}{9} - \frac{256}{9}) - 64 = 0$

$3(x^2 + \frac{32}{3}x + \frac{256}{9}) - \frac{256}{3} - 64 = 0$

$3(x + \frac{16}{3})^2 = 64 + \frac{256}{3}$

$3(x + \frac{16}{3})^2 = \frac{448}{3}$

$x + \frac{16}{3} = \pm\sqrt{\frac{448}{9}}$

$x = -\frac{16}{3} \pm \frac{\sqrt{448}}{3}$

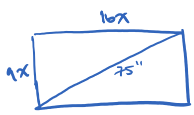
$x = \frac{-16}{3} \pm \frac{8\sqrt{7}}{3}$

$\boxed{x_1 = \frac{-16}{3} + \frac{8\sqrt{7}}{3}}$

$x_2 = \frac{-16}{3} - \frac{8\sqrt{7}}{3}$ ← CAN'T HAVE NEGATIVE VALUE

12. Jason bought a 75" television at Costco. He knows that the screen aspect ratio is 16:9 [width to height]. Besides the screen, there is also a uniform border of 2" around. What is the width of the TV?

① THE BORDER IS NOT PART OF THE SCREEN.



$(16x)^2 + (9x)^2 = (75)^2$

$256x^2 + 81x^2 = 75^2$

$337x^2 = 75^2$

$x^2 = \frac{75^2}{337}$

$x^2 = 16.6913947...$

$x = 4.0855$

② WIDTH $= 9x + 2$

$= 9(4.0855) + 2$

$= 38.769$"

LENGTH $= 16(4.0855) + 2$

$= 67.368$

